

## RELAXATION OSCILLATIONS DURING GAS COMBUSTION IN FURNACES

O. G. Roginskii

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The author examines two models of nonacoustic relaxation oscillations during vibratory combustion of a gas; these correspond to the two extreme cases of furnace systems  $G = \text{idem}$  and  $p_f = \text{idem}$ .

It is known [1] that excitation of oscillations in combustion systems is connected with perturbations of the heat supply and perturbations of the burning rate. Both these and other perturbations are connected, in their turn, with oscillations of the flow velocity and pressure. Two extreme cases are possible: 1) perturbations of the heat supply (or burning rate) related to oscillations of the flow velocity of the gas-air mixture, the mass flow over all sections of the system being the same at all times ( $G = \text{idem}$ ); 2) perturbations of the heat supply (or burning rate) related to pressure oscillations in the furnace, the pressure being the same, at all times and at all points in the furnace ( $p_f = \text{idem}$ ).

It is then expedient to examine systems of type  $G = \text{idem}$  and  $p_f = \text{idem}$  as convenient theoretical models. In addition, in this paper we shall examine only oscillations associated with perturbations in the heat supply.

As a system of type  $G = \text{idem}$  let us examine a furnace with an injection gas burner equipped with a nozzle in the form of a plate combustion stabilizer.

The basic equation for joint operation of burner and furnace in the transient regime, allowing for the inertia head of the injector, may be written in the form

$$U \frac{dz}{dt} = \Delta p_i^* - \Delta p_n^* + \Delta p_f^*, \quad (1)$$

where

$$U = \frac{2}{\rho_g \omega_g^2} \left[ \frac{\rho_a}{\rho_g} (F_j \rho_g \omega_g) \int_0^L \frac{dx}{F} \right]; \quad z = \frac{Q_a}{Q_g};$$

$$\Delta p_i^* = \frac{2\Delta p_i}{\rho_g \omega_g^2}; \quad \Delta p_n^* = \frac{2\Delta p_n}{\rho_g \omega_g^2}; \quad \Delta p_f^* = \frac{2\Delta p_f}{\rho_g \omega_g^2};$$

$$\Delta p_i = p_i - p_0; \quad \Delta p_n = p_n - p_f; \quad \Delta p_f = p_0 - p_f.$$

In deriving (1) it was assumed that the flow rate of mixture in the system is the same at each instant of time, while the flow rate of gas is constant in time. Moreover, we shall henceforth consider, for simplicity, that the underpressure in the furnace is constant ( $\Delta p_f = \text{const}$ ).

The heads  $\Delta p_i^*$  and  $\Delta p_n^*$  are functions of the injection coefficient [2]:

$$\Delta p_i^* = R - S'z - Tz^2, \quad (2)$$

$$\Delta p_n^* = D[Ez^2 + (1 + E)z + 1]. \quad (3)$$

A plate stabilizer nozzle is a system of plates connected by rods, each of which acts as a stabilizer in the form of a poorly streamlined body. The flame is confined to the turbulent wake of the rods.

The bundle of plates prevents the flame jumping inside the burner.

Tests show that, at small mixture velocities, the flame "sits" on the entire stabilizer—rods and plates. At increased velocities the flame gradually separates from the plates, and is finally confined to the rods alone. With further increase in mixture velocity the flame separates first from the extreme rods, and then from rods located closer to the center, until it is completely detached. Thus several sharply defined stages of separation may be observed, the pattern described being observed at various values of the injection coefficient (and hence of the air-fuel ratio). When the mixture velocity is reduced, stages of "reattachment" develop in reverse order. In this context two very important circumstances should be noted, which allow a model of the relaxation oscillations to be constructed for the case examined: transition to each successive stage of separation of the flame is accompanied by a downward pressure jump ahead of the stabilizer (i. e., in the head  $\Delta p_n$ ), just as the reverse transition is accompanied by an upward pressure jump; the bound-

aries of the stages of separation and of the corresponding stages of "reattachment" of the flame have various locations depending on the injection coefficient and the gas-air mixture velocity (Fig. 1).

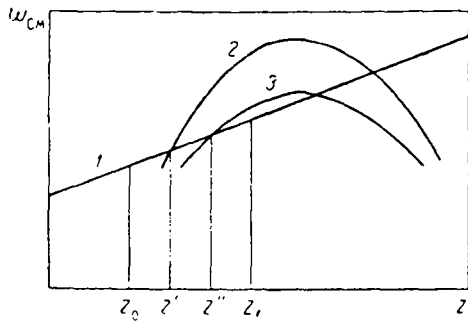


Fig. 1. Boundaries of the stages of separation (2) and "reattachment" (3), and determination of points of separation and "reattachment" ( $1 - w_m = (m/r) w_g (1 + z)$ ).

The first may be explained by variation of the thermal resistance of the flame, which is part of the resistance of the nozzle. Due to the fact that there is a sharp increase in temperature at the flame front, the flame possesses thermal resistance, which falls with increase in the degree of separation from the stabilizer. This resistance is commensurable with that of the "cold" nozzle (stabilizer).

The second is evidently connected with the laws of flame stabilization behind poorly streamlined bodies. If, following Vulis [3], we treat the conditions of flame stability and instability as conditions of ignition and quenching near the recirculation zone behind the stabilizer, the noncoincidence of the flame separation and "reattachment" boundaries may be explained by the well-known noncoincidence of the quenching and ignition points. This lack of coincidence for the flame separation and "reattachment" boundaries (hysteresis) is due to static bistability of the system in the interval between these boundaries.

We shall assume that separation and "reattachment" correspond to constant values  $A_1$  and  $A_2$  of the hydraulic resistance coefficient, where  $A_1 < A_2$ . This gives us the following equations for the nozzle:

$$\Delta p_n^* = D_1 [Ez^2 + (1 + E)z + 1], \quad (4)$$

$$\Delta p_n^* = D_2 [Ez^2 + (1 + E)z + 1]. \quad (5)$$

Here  $D_1 \approx A_1$ ,  $D_2 \approx A_2$ , and  $D_1 < D_2$ .

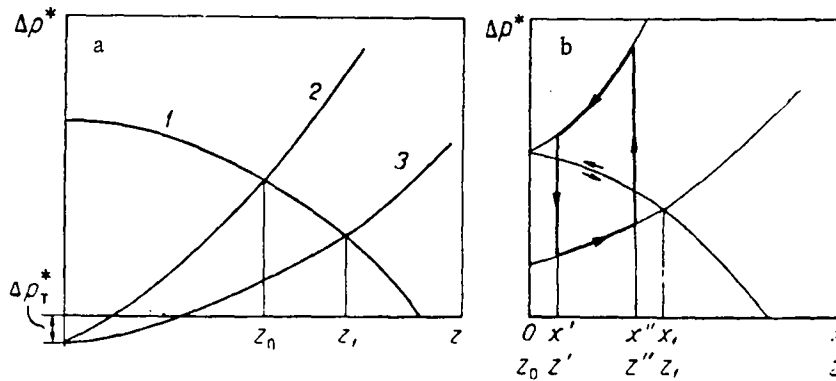


Fig. 2. a) Characteristics of injector ( $1 - \Delta p_i^* = f(z)$ ) and nozzle ( $2 - \Delta p_n^* = f(z)$ ) for  $A = A_2$ ;  $3 - \Delta p_n^* = f(z)$  for  $A = A_1$ , and b) diagram of self-oscillation of the system  $G = \text{idem}$ .

The characteristics of the injector, according to (2), and of the nozzle, according to (4) and (5), are shown graphically in Fig. 2a.

We introduce the new variable

$$x = z - z_0, \quad (6)$$

where  $z_0$  is the injection coefficient corresponding to the point of intersection of the injector characteristic (2) and the nozzle characteristic (5). Using (6), we transform (2), (4), and (5) and substitute in the original equation (1).

Omitting the simple intermediate steps, we have: if the flame is separated from the stabilizer,

$$\dot{x} = a - 2b_1 x - c_1 x^2, \quad (7)$$

if the flame is "sitting" on the stabilizer,

$$\dot{x} = -2b_2 x - c_2 x^2, \quad (8)$$

where

$$\begin{aligned}
 a &= (D_2 - D_1) [Ez_0^2 + (1 + E)z_0 + 1] U^{-1}, \\
 2b_1 &= [S + 2z_0(T + D_1E) + D_1(1 + E)] U^{-1}, \\
 2b_2 &= [S + 2z_0(T + D_2E) + D_2(1 + E)] U^{-1}, \\
 c_2 &= (T + D_1E) U^{-1}, \quad c_2 = (T + D_2E) U^{-1}.
 \end{aligned}
 \tag{9}$$

After separating the variables and integrating, we can write the solution of (7) as

$$-\frac{1}{2\sqrt{ac_1 + b_1^2}} \ln \frac{\sqrt{ac_1 + b_1^2} - b_1 - c_1x}{\sqrt{ac_1 + b_1^2} + b_1 + c_1x} = t + \text{const.}
 \tag{10}$$

The solution of (8), obtained similarly, may be written as

$$-\frac{1}{2b_2} \ln \frac{x}{c_2x + 2b_2} = t + \text{const.}
 \tag{11}$$

Equation (10) gives a function for  $x$  that grows exponentially with time, and (11) a decreasing one. To obtain  $x$  as a continuous function of time  $t$ , the solutions of (10) and (11) must be "joined." This may be done as follows.

From the theory of injectors it is known that the connection between mixture velocity and injection coefficient [2] is

$$w_m = mw_g(1 + z)/r,
 \tag{12}$$

where  $m = F_j/F_{th}$ ;  $r = F_d/F_{th}$ .

For constant burner geometry ( $m = \text{const}$ ,  $r = \text{const}$ ) and given gas velocity ( $w_g = \text{const}$ ), in the  $w_m$ ,  $z$  plane, Eq. (12) is represented by a straight line, along which these burner parameters vary. This curve may intersect the known curves of flame separation and "reattachment." Our interest here is in the points of intersection lying to the left of the maxima, and corresponding to values of the injection coefficient equal to  $z'$  and  $z''$  (Fig. 1). Let these values lie in the range  $z_1 - z_0$ . Then the parts of the exponentials described by (10) and (11) must lie in the interval  $z'' - z'$  (or, which is the same thing, in the interval  $x'' - x'$ ), i. e., the exponentials must intersect at the points corresponding to  $x = x'$  and  $x = x''$ .

In this way we obtain an oscillogram of the oscillating relaxation process. Each oscillation cycle consists of two stages. Determining the duration of the first stage from (10) and the duration of the second from (11), it is easy to find the period of oscillation as

$$\begin{aligned}
 T_{osc} = & \frac{1}{2\sqrt{ac_1 + b_1^2}} \ln \frac{(\sqrt{ac_1 + b_1^2} - b_1 - c_1x')(\sqrt{ac_1 + b_1^2} + b_1 + c_1x'')}{(\sqrt{ac_1 + b_1^2} - b_1 - c_1x'')(\sqrt{ac_1 + b_1^2} + b_1 + c_1x')} + \\
 & + \frac{1}{2b_2} \ln \frac{x''(c_2x' + 2b_2)}{x'(c_2x'' + 2b_2)}.
 \end{aligned}
 \tag{13}$$

Figure 2b shows a diagram of the oscillatory process, which we can now describe in the following terms. For some small value of the injection coefficient the flame is separated (fully or partially) from the stabilizer, the injector pressure being greater than the resistance of the stabilizer and flame. Therefore the injection coefficient increases, and the increase continues until at some value of the injection coefficient  $z = z''$ , the flame "reattaches" itself to the stabilizer\*. Then the nozzle operating point jumps from one characteristic to the other, which corresponds to higher resistance. The injector pressure is now less than the resistance of the stabilizer and flame, so that the injection coefficient begins to fall. At some small value  $z = z'$  flame separation sets in, resulting in a downward jump in nozzle resistance. The operating point again lies on the less steep characteristic, and the process is repeated.

High-speed motion-picture photography of the process shows that low-frequency vibratory burning involves periodic flame separation.

Under what conditions is self-oscillation impossible?

From the formal point of view, these conditions may be found by putting  $T_{osc} = \infty$ . It follows from (13) that then at least one of the following conditions must be satisfied:

\* "Reattachment" of the separated flame may occur at any degree of separation (partial or full) and is impossible only when the flame collapses.

$$\sqrt{ac_1 + b_1^2} - b_1 - c_1 x'' = 0$$

or

$$x' = 0.$$

The first condition leads to the requirement  $z'' = z_1$ , and the second gives  $z' = z_0$ .

Thus self-oscillation is impossible when

$$z' \leq z_0, \quad z'' \geq z_1.$$

Physically, this means that the system must be able to realize equilibrium conditions corresponding to  $z_0$  or  $z_1$ . In other words, the range of operating conditions of the burner should lie (at least, in part) within the bistable zone or the hysteresis region. This can be done by a suitable choice of the design and operating parameters of the system.

Tests show that when the construction of the stabilizer is changed the boundary curves may be displaced upwards, increasing the region of stable operation of the burner. When the injector parameters are varied (for example, the parameter  $k = F_a/F_{th}$ ), the boundary curves of separation and "reattachment" are displaced, the straight line  $w_m = f(z)$  cannot intersect both curves at once to the left of the maximum, and hence oscillations become impossible. (It is easily shown that intersection of the two curves to the right of the maximum does not give oscillations.)

Let us examine a system of the type  $p_f = idem$  as an example of a furnace with a mixing burner (a burner with a forced supply of gas and air).

The basic equation for this case is the heat balance equation in the transient regime

$$dQ/dt = q_1 - q_2, \quad (14)$$

where

$$Q = c_{vf} G_f T_f; \quad q_1 = G_g (Q_{fl}^{\xi} / \gamma_g + c_{pg} T_0) + G_a c_{pa} T_0; \\ q_2 = G_{fl} c_{pfl} T_{fl} + k_f S_f (T_f - T_0).$$

We shall consider, for simplicity, that combustion occurs instantaneously at the burner outlet, and that the thermodynamic state of the combustion products at each instant of time is identical over the whole volume of the furnace. Then the equation of state for the furnace gases is written in the form

$$p_f V_f = G_f R_f T_f. \quad (15)$$

Further, we shall write the mass flow characteristics of the burner (gas and air) and of the flue:

$$G_g = \nu_g F_g \sqrt{2g \gamma_g (p_g - p_f)}, \\ G_a = \nu_a F_a \sqrt{2g \gamma_a (p_a - p_f)}, \\ G_{fl} = \nu_{fl} F_{fl} \sqrt{2g \gamma_{fl} (p_f - p_{fl})}. \quad (16)$$

We assume that

$$c_{pfl} = c_{pf}, \quad T_{fl} = T_f, \quad \gamma_{fl} = \gamma_f = p_f / R_f T_f \approx p_{r0} / R_f T_f. \quad (17)$$

The completeness of combustion  $\xi$  and the temperature  $T_f$  in the furnace depend appreciably on the burning conditions. It is evident that when the pressure in the furnace increases, the pressure drops in the burner fall (in the general case, by different amounts), while the pressure drop between the furnace and the flue increases. This causes an excess of heat removal over heat supply, and, if the pressure excess in the furnace is large enough, it may lead to quenching of the flame. Conversely, a decrease of pressure in the furnace causes an excess of heat supplied over heat removed. When the excess pressure in the furnace is small enough (in particular, when there is an underpressure), and when there is a hot source (a glowing hot furnace lining or hot combustion products that have not been exhausted), ignition of the mixture occurs.

As before, we note two factors which determine the possibility of constructing models of the relaxation oscillations for the given case: ignition and quenching are accompanied by discontinuous changes in the temperature in the furnace and in the completeness of combustion, and occur at various values of the furnace pressure.

The first is obvious. The second follows from the fact that the ignition and quenching points do not coincide.

Let the temperatures in the furnace after ignition and quenching be, respectively,  $T_f$  and  $T_f$ , and the coefficients of completeness of combustion  $\xi_1$  and  $\xi_2 = 0$ .

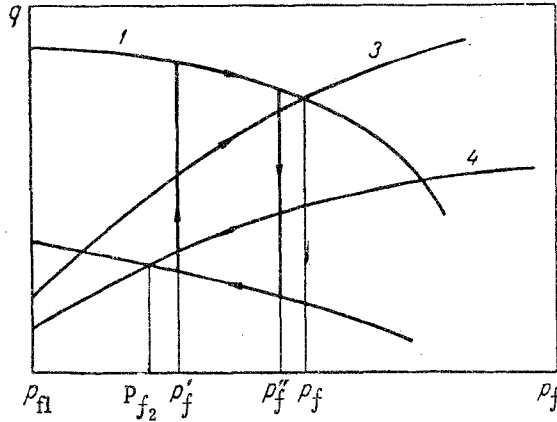


Fig. 3. Characteristics of heat supply and removal, and diagram of self-oscillations of the system  $p_f = \text{idem}$ : 1 and 2)  $q_1$  for  $T_f = T_{f1}$  and for  $T_f = T_{f2}$ ; 3 and 4)  $q_2$  for  $T_f = T_{f1}$  and for  $T_f = T_{f2}$ .

We introduce the new variable

$$y = p_f - p_{fl} \quad (21)$$

where  $p_{fl}$  is the absolute pressure in the flue downstream of the damper, which for simplicity we shall consider constant. In addition, we designate

$$p_g - p_{fl} = a, \quad p_a - p_{fl} = b. \quad (22)$$

Using (21) and (22), we transform (16) and substitute it in the expressions for  $q_1$  and  $q_2$  and also into the original equation (14). Using (15) and (17), after intermediate steps, we obtain: when there is combustion

$$\dot{y} = K'_1 \sqrt{a-y} + L' \sqrt{b-y} - M_1 \sqrt{y} - N'_1, \quad (23)$$

when there is no combustion

$$\dot{y} = K'_2 \sqrt{a-y} + L' \sqrt{b-y} - M_2 \sqrt{y} - N'_2, \quad (23) \text{ (cont'd)}$$

where

$$\left. \begin{aligned} K'_1 &= \mu_g F_g \sqrt{2g \gamma_g R_g} \frac{Q_n^p \xi_1 / \gamma_g + c_{pg} T_0}{c_{vf} V_f}; \\ K'_2 &= \mu_g F_g \sqrt{2g \gamma_g} R_f c_{pg} T_0 / c_{vf} V_f; \\ L' &= \mu_a F_a \sqrt{2g \gamma_a} R_f c_{pa} T_0 / c_{vf} V_f; \\ M_1 &= \mu_{fl} F_{fl} \sqrt{2g p_{f0}} R_f T_{f1} c_{pf} / c_{vf} V_f; \\ M_2 &= \mu_{fl} F_{fl} \sqrt{2g p_{f0}} R_f T_{f2} c_{pf} / c_{vf} V_f; \\ N'_1 &= k_f S_f (T_{f1} - T_0) R_f / c_{vf} V_f; \quad N'_2 = k_f S_f (T_{f2} - T_0) R_f / c_{vf} V_f. \end{aligned} \right\} \quad (23')$$

In (23) the variables are separated, but integration presents great computing difficulties. The problem may be simplified by expanding  $\sqrt{a-y}$  and  $\sqrt{b-y}$  by Newton's binomial theorem and restricting attention (to no special detriment) to the linear terms

$$\sqrt{a-y} \approx \sqrt{a} - y/2\sqrt{a}, \quad \sqrt{b-y} \approx \sqrt{b} - y/2\sqrt{b}.$$

Then instead of (23) we may write

$$\begin{aligned} \dot{y} &= L_1 - M_1 \sqrt{y} - N_1 y, \\ \dot{y} &= L_2 - M_2 \sqrt{y} - N_2 y, \end{aligned} \quad (24)$$

Then we obtain the following expressions for  $q_1$ ,  $q_2$ , and  $G_{fl}$ :

$$\text{when } \xi = \xi_1 \quad q_1 = G_g \left( \frac{Q_n^p}{\gamma_g} \xi_1 + c_{pg} T_0 \right) + G_a c_{pa} T_0,$$

$$\text{when } T_f = T_{f1} \quad \begin{cases} q_2 = G_{fl} c_{pf} T_{f1} + k_f S_f (T_{f1} - T_0), \\ G_{fl} = \mu_{fl} F_{fl} \sqrt{2g \gamma_{f1} (p_f - p_{fl})}, \end{cases} \quad (18)$$

$$\text{when } \xi = \xi_2 = 0 \quad q_1 = G_g c_{pg} T_0 + G_a c_{pa} T_0, \quad (19)$$

$$\text{when } T_f = T_{f2} \quad \begin{cases} q_2 = G_{fl} c_{pf} T_{f2} + k_f S_f (T_{f2} - T_0), \\ G_{fl} = \mu_{fl} F_{fl} \sqrt{2g \gamma_{f2} (p_f - p_{fl})}. \end{cases} \quad (20)$$

Figure 3 presents curves of variation of heat input and output as functions of the pressure in the furnace according to (20), (21), (23), (24) with account for (16), (17), (22), and (25).

where

$$L_1 = K_1' \sqrt{a} + L' \sqrt{b} - N_1'; \quad L_2 = K_2' \sqrt{a} + L' \sqrt{b} - N_2';$$

$$N_1 = \frac{1}{2} \left( \frac{K_1'}{\sqrt{a}} + \frac{L'}{\sqrt{b}} \right); \quad N_2 = \frac{1}{2} \left( \frac{K_2'}{\sqrt{a}} + \frac{L'}{\sqrt{b}} \right). \quad (25)$$

After separation of the variables and integration, the solution of (24) takes the general form

$$-\frac{1}{N} \ln (L - M \sqrt{y} - Ny) -$$

$$-\frac{M}{N \sqrt{4LN + M^2}} \ln \frac{M + \sqrt{4LN + M^2} + 2N \sqrt{y}}{M - \sqrt{4LN + M^2} + 2N \sqrt{y}} = t + \text{const.} \quad (26)$$

For values of the coefficients of (24) and (25) with subscript 1, (26) gives a function for  $y$  that increases with time, and for subscript 2 a decreasing one. "Joining" of the solutions to obtain  $y$  as a continuous function of  $t$  is done in the same way as before.

From (16) and the relation

$$\alpha = G_g \gamma_a / G_a \gamma_g L_0,$$

it is not difficult to obtain the excess pressure in the furnace (over that in the flue) and  $\alpha$ :

$$p_f - p_{fl} = \frac{(p_g - p_{fl}) \alpha^2 / k^2 - (p_a - p_{fl})}{\alpha^2 / k^2 - 1}, \quad (27)$$

where

$$k = \frac{1}{L_0} \frac{\rho_a}{\rho_g} \frac{F_a}{F_g} \sqrt{\frac{\gamma_g}{\gamma_a}}.$$

For fixed burner geometry ( $k = \text{const}$ ) and given pressure conditions ( $p_f = \text{const}$ ,  $p_a = \text{const}$ ,  $p_{fl} = \text{const}$ ), (27) describes a curve in the plane of parameters ( $p_f - p_{fl}$ ) along which these parameters vary. This curve may intersect certain curves of ignition and quenching. Let the points of intersection correspond to the values  $y'$  and  $y''$ . If these values lie in the interval  $y_1 - y_2$ , then segments of the exponentials described by (26) must lie in the interval  $y'' - y'$ , or, which is the same thing, the exponentials will intersect at points corresponding to  $y = y'$  and  $y = y''$ . As before, each cycle consists of two stages (Fig. 3). We can at once write the expression for the period of oscillation:

$$T_{\text{osc}} = \frac{1}{N_1} \ln \left( \frac{L_1 - M_1 \sqrt{y'} - N_1 y'}{L_1 - M_1 \sqrt{y''} - N_1 y''} \right) + \frac{M_1}{N_1 \sqrt{4L_1 N_1 + M_1^2}} \times$$

$$\times \ln \left[ \left( \frac{M_1 + \sqrt{4L_1 N_1 + M_1^2} + 2N_1 \sqrt{y'}}{M_1 - \sqrt{4L_1 N_1 + M_1^2} + 2N_1 \sqrt{y'}} \right) \times \right.$$

$$\times \left. \left( \frac{M_1 + \sqrt{4L_1 N_1 + M_1^2} + 2N_1 \sqrt{y''}}{M_1 - \sqrt{4L_1 N_1 + M_1^2} + 2N_1 \sqrt{y''}} \right)^{-1} \right] +$$

$$+ \frac{1}{N_2} \ln \left( \frac{L_2 - M_2 \sqrt{y''} - N_2 y''}{L_2 - M_2 \sqrt{y'} - N_2 y'} \right) + \frac{M_2}{N_2 \sqrt{4L_2 N_2 + M_2^2}} \times$$

$$\times \ln \left[ \left( \frac{M_2 + \sqrt{4L_2 N_2 + M_2^2} + 2N_2 \sqrt{y''}}{M_2 - \sqrt{4L_2 N_2 + M_2^2} + 2N_2 \sqrt{y''}} \right) \times \right.$$

$$\times \left. \left( \frac{M_2 + \sqrt{4L_2 N_2 + M_2^2} + 2N_2 \sqrt{y'}}{M_2 - \sqrt{4L_2 N_2 + M_2^2} + 2N_2 \sqrt{y'}} \right)^{-1} \right]. \quad (28)$$

The oscillatory process is as follows. At some small value of the pressure in the furnace, due to the large difference between the gas and air pressures, on the one hand, and the furnace pressure, on the other, gas and air enter the furnace in sufficient quantity and in the required ratio for burning. Since the flow rate out of the furnace is small (the difference in pressure between furnace and flue is small), and the heat supply is greater than the heat loss, the pressure

in the furnace increases. The increase continues until the flame is quenched at some pressure  $p_f^*$  in the furnace, owing either to reduction of the gas supply or to a change in  $\alpha$  making burning impossible. Then a sharp fall in temperature occurs in the furnace, and a transition to new curves of heat supply and removal giving increased heat supply, which leads to a drop in furnace pressure. At some small value of the pressure  $p_f'$  ignition occurs due to the change in gas flow rate and  $\alpha$ , and the process repeats itself.

The conditions for which self-oscillation are impossible, as in the previous case, are determined from  $T_{osc} = \infty$ . According to (28), this means

$$L_1 - M_1 \sqrt{y''} - N_1 y'' = 0; \quad L_2 - M_2 \sqrt{y'} - N_2 y' = 0.$$

The first condition is none other than an approximate form of the exact heat balance equation for combustion [cf. (26) and (23)]:

$$K_1' \sqrt{a - y''} + L' \sqrt{b - y''} - M_1 \sqrt{y''} - N_1' = 0,$$

which is possible only when  $y'' = y_1$ , i. e., when  $p_f'' = p_{f_1}$ . An analogous analysis of the second condition gives  $p_f' = p_{f_2}$ .

Therefore, self-oscillation is impossible when

$$p_f' \leq p_{f_2}, \quad p_f'' \geq p_{f_1}.$$

As in the previous case, these conditions mean that at least part of the range of furnace operating conditions should lie within the bistable zone, which can be ensured by an appropriate choice of design and operating parameters.

#### NOTATION

G—mass flow rate; Q—volume flow rate;  $\gamma$ —specific weight; z—volume injection coefficient;  $\Delta p_1^*$ ,  $\Delta p_n^*$ —dimensionless pressure head created by injector and operating in nozzle (from the static characteristic), respectively;  $\Delta p_f^*$ —dimensionless underpressure in furnace; R, S', T—known coefficients, functions of injector geometry, hydraulic resistance along injector channel, and the parameter  $E = \rho_a / \rho_g$ ; D—a coefficient, a function of the injector geometry and the hydraulic resistance of the nozzle (it depends appreciably on the burning conditions);  $\rho$ —density; T—temperature; p—pressure; w—velocity;  $c_p$ —specific heat at constant pressure;  $c_v$ —specific heat at constant volume; G'—mass change; V—volume; R—gas constant;  $\alpha$ —fuel-air ratio;  $\xi$ —coefficient of completeness of combustion; k—heat transfer coefficient;  $Q_1^p$ —lower heat of combustion of gas;  $L_0$ —volume of air required for complete combustion of 1 m<sup>3</sup> of gas at NTP;  $\mu$ —mass flow rate coefficient; S—heat transfer surface; F—flow section area; Q,  $q_1$ , and  $q_2$ —amount of heat in one furnace charge, introduced by gas and air and removed from furnace by heated combustion products and by heat removal, respectively;  $p_{fl}$ —absolute pressure in flue downstream of damper. Subscripts: g—gas; a—air; fl—flue; f—furnace; j—jet; th—throat; d—diffuser; m—mixture; 0—under initial conditions.

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"Mosgazproekt" Planning Institute, Moscow